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A Nonlinear Model Inversion Method for Joint System Parameter, Noise, and Input Identification of Civil Structures

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Abstract

This paper presents a framework for nonlinear system identification of civil structures using sparsely measured dynamic output response of the structure. Using a sequential maximum likelihood estimation (MLE) approach, the unknown FE model parameters, the measurement noise variances, and the input ground acceleration time histories are estimated jointly. This approach requires the computation of FE response sensitivities with respect to the unknown FE model parameters (i.e., FE parameter sensitivities) as well as the FE response sensitivities with respect to the values of the input ground acceleration at every time step (i.e., FE input sensitivities). The FE parameter and input sensitivities are computed using the direct differentiation method (DDM). The presented output-only nonlinear FE model updating method is validated using the numerically simulated seismic response of a realistic three-dimensional five-story reinforced concrete building structure. The simulated building responses to a horizontal bi-directional seismic excitation is contaminated with artificial measurement noise and used to estimate the unknown FE model parameters characterizing the nonlinear material constitutive laws of the reinforced concrete, as well as the root mean square of the measurement noise at each measurement channel, and the full time history of the seismic base acceleration. The method presented in this paper provides a powerful framework for structural system and damage identification of civil structures, when the input excitations are not measured, are partially measured, or the measured input excitations are erroneous.

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Keywords: Nonlinear Finite Element Model; Model Updating; Joint Input and System Identification; Input Estimation; Bayesian Inference; Damage Identification; Structural Health Monitoring.

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1. Introduction

Given the measured dynamic response of a civil structure during an earthquake event, our goal is to detect, localize, and quantify the state of damage in the structural system. We will pursue this objective using an output-only nonlinear finite element (FE) model updating approach. The nonlinear FE model of a civil structure depends on a set of unknown model parameters including but not limited to inertial properties, gravity loading, damping parameters, and parameters characterizing the nonlinear material constitutive laws. The goal of a nonlinear FE model updating procedure is to estimate the most probable values of these model parameters and quantify their estimation uncertainties. A high-fidelity mechanics-based nonlinear FE model with updated model parameters is able to capture the complex damage/failure mechanisms in the structural system of interest. Nevertheless, measuring accurately the complete earthquake excitation input to real-world civil structures is often difficult. For example, measuring the seismic input excitation in the case of underground structures, buildings with subterranean levels, and bridges with deep-water piers can be challenging, if not impossible. Hence, we present a framework for updating nonlinear mechanics-based FE models using only the measured dynamic response of the civil structure. The framework is based on a sequential maximum likelihood estimation (MLE) approach to estimate jointly the unknown FE model parameters, the measurement noise variances, and the input ground acceleration time histories.

2. Problem Formulation

The time-discretized equation of motion of a nonlinear FE model at time step $\boldsymbol{i}$ ($\boldsymbol{i}=1\rightarrow\boldsymbol{k}$, where k denotes the total number of time steps) is expressed as

$$\mathbf{M}(\boldsymbol{\theta})\ \ddot{\mathbf{q}}_i(\boldsymbol{\theta})+\mathbf{C}(\boldsymbol{\theta})\ \dot{\mathbf{q}}_i(\boldsymbol{\theta})+\boldsymbol{\gamma}_i\left(\mathbf{q}_{1:i}(\boldsymbol{\theta}),\boldsymbol{\theta}\right)=\mathbf{f}_i(\boldsymbol{\theta}) \tag{1}$$

where $\mathbf{M}(\boldsymbol{\theta})\in\mathbb{R}^{n_{DOF}\times n_{DOF}}$ = mass matrix; $\mathbf{C}(\boldsymbol{\theta})\in\mathbb{R}^{n_{DOF}\times n_{DOF}}$ = damping matrix; $\boldsymbol{\gamma}_i\left(\mathbf{q}_{1:i}(\boldsymbol{\theta}),\boldsymbol{\theta}\right)\in\mathbb{R}^{n_{DOF}\times 1}$ = history-dependent (or path-dependent) internal resisting force vector; $\mathbf{q}_i(\boldsymbol{\theta}),\dot{\mathbf{q}}_i(\boldsymbol{\theta}),\ddot{\mathbf{q}}_i(\boldsymbol{\theta})\in\mathbb{R}^{n_{DOF}\times 1}$ = nodal displacement, velocity, and acceleration response vectors, respectively; $\boldsymbol{\theta}\in\mathbb{R}^{n_{\boldsymbol{\theta}}\times 1}$ = vector of unknown FE model parameters; $\mathbf{f}_i(\boldsymbol{\theta})\in\mathbb{R}^{n_{DOF}\times 1}$ = dynamic load vector; and n_{DOF} = number of degrees of freedom. In the case of uniform (or rigid base) seismic excitation, $\mathbf{f}_i(\boldsymbol{\theta})=-\mathbf{M}(\boldsymbol{\theta})\mathbf{L}\ddot{\mathbf{u}}_i^g$ where $\mathbf{L}\in\mathbb{R}^{n_{DOF}\times n_{\ddot{\mathbf{u}}^g}}$ = base acceleration influence matrix, and $\ddot{\mathbf{u}}_i^g\in\mathbb{R}^{n_{\ddot{\mathbf{u}}^g}\times 1}$ denotes the seismic input ground acceleration vector. Using a recursive numerical integration rule, such as the Newmark-beta method [1], Eq. (1) is reduced to a nonlinear vector-valued algebraic equation that can be solved recursively and iteratively for the nodal displacement response vector at each time step. The nodal velocity and acceleration response vectors can then be derived from the nodal displacements. In general, the response of a FE model, corresponding to the measured response of the structure of interest, can be expressed as a linear or nonlinear function of the nodal displacement, velocity, and/or acceleration response vectors at each time step. Denoting the response quantity predicted by the FE model at time step i by $\hat{\mathbf{y}}_i\in\mathbb{R}^{n_{\mathbf{y}}\times 1}$, it follows that

$$\hat{\mathbf{y}}_i=\mathbf{h}_i\left(\boldsymbol{\theta},\ddot{\mathbf{u}}_{1:i}^g\right) \tag{2}$$

where $\mathbf{h}_i(\ldots)$ is the nonlinear response function of the FE model at time step i. The measured response vector of the structure, $\mathbf{y}_i$, can be related to the FE predicted response, $\hat{\mathbf{y}}_i$, through

$$\mathbf{v}_i\left(\boldsymbol{\theta},\ddot{\mathbf{u}}_{1:i}^g\right)=\mathbf{y}_i-\hat{\mathbf{y}}_i\left(\boldsymbol{\theta},\ddot{\mathbf{u}}_{1:i}^g\right) \tag{3}$$

in which $\mathbf{v}_i\in\mathbb{R}^{n_{\mathbf{y}}\times 1}$ is the simulation error vector and accounts for the misfit between the measured and predicted responses of the structure. In the absence of modeling uncertainties, $\mathbf{v}_i$ in Eq. (3) accounts only for the measurement noise. Furthermore, it is assumed here that the measurement noises are stationary, zero-mean, and independent

Gaussian white noise processes (i.e., statistically independent across time and measurement channels), i.e, $\mathbf{v}_i \sim N(\mathbf{0},\mathbf{R}), \forall i$, where $\mathbf{R}$ denotes the simulation error covariance matrix, which is diagonal [2].

In the proposed output-only structural system parameters and input identification method, the FE model parameter vector ($\boldsymbol{\theta}$) and the discrete values of the seismic input ground acceleration time history ($\ddot{\mathbf{u}}^g_{1:k}$) are modeled as random variables. The unknown FE model parameters and the ground acceleration time history are estimate jointly such that their joint posterior PDF given the measured response of the structure is maximized. Assuming a uniform (non-informative) prior PDFs, maximizing the posterior PDF reduces to a maximum likelihood (ML) estimation problem as [3]

$$\left(\hat{\boldsymbol{\theta}},\hat{\ddot{\mathbf{u}}}^g_{1:k}\right)_{\mathrm{ML}} = \underset{\left(\boldsymbol{\theta},\ddot{\mathbf{u}}^g_{1:k}\right)}{\arg\max}\; p\left(\mathbf{y}_{1:k}\,\middle|\,\boldsymbol{\theta},\ddot{\mathbf{u}}^g_{1:k}\right) \tag{4}$$

in which $\mathbf{y}_{1:k} = \left[\mathbf{y}_1^T,\mathbf{y}_2^T,\ldots,\mathbf{y}_k^T\right]^T$ = time history of the measured response of the structure. As proposed in [2], to enhance the robustness of the parameter estimation procedure, the variance of the components of the simulation error vector (i.e., the diagonal entries of matrix $\mathbf{R}$) are also treated as unknowns and estimated jointly with the other unknowns through an extended ML estimation framework. This results in a joint system parameter, input, and noise identification problem. The diagonal entries of the covariance matrix $\mathbf{R}$ are stacked in a row vector called the simulation error variance vector $\mathbf{r} = \{r_j\}, 1 \le j \le n_y$, where r_j is the j^{th} diagonal entry of $\mathbf{R}$. The extended ML estimation problem results in the following nonlinear optimization problem.

$$\left(\hat{\boldsymbol{\theta}},\hat{\ddot{\mathbf{u}}}^g_{1:k},\hat{\mathbf{r}}\right) = \underset{\left(\boldsymbol{\theta},\ddot{\mathbf{u}}^g_{1:k},\mathbf{r}\right)}{\arg\min}\; J\left(\mathbf{y}_{1:k},\boldsymbol{\theta},\ddot{\mathbf{u}}^g_{1:k},\mathbf{r}\right) \tag{5}$$

$$J\left(\mathbf{y}_{1:k},\boldsymbol{\theta},\ddot{\mathbf{u}}^g_{1:k},\mathbf{r}\right) = \left(\frac{k}{2}\sum_{j=1}^{n_y}\ln\left(r_j\right) + \frac{1}{2}\sum_{j=1}^{n_y}\sum_{i=1}^{k}\frac{\left(y_{ij} - h_{ij}\left(\boldsymbol{\theta},\ddot{\mathbf{u}}^g_{1:i}\right)\right)^2}{r_j}\right) \tag{6}$$

where $J(\ldots)$ = optimization objective function, y_{ij} = j^{th} component of the measured structural response vector at time step i, and similarly h_{ij} = j^{th} component of the FE predicted structural response vector at time step i. The nonlinear optimization problem defined in Eqs. (5)-(6) can be solved using gradient-based optimization methods, which require the computation of the gradient vector of the objective function with respect to the estimation parameters. The gradient of the objective function depends on the rate of variation (or sensitivity) of the FE predicted response with respect to the FE model parameter vector $\boldsymbol{\theta}$ and the base acceleration vector $\ddot{\mathbf{u}}^g_{1:k}$. In this study, the FE response sensitivities are computed using the direct differentiation method (DDM), which is based on the exact (consistent) differentiation of the FE numerical scheme with respect to the sensitivity parameters [4-7]. More details on how to derive the FE response sensitivity with respect to the uniform base excitation can be found in [8].

A sequential estimation approach is proposed here to solve the extended ML problem. In this approach, the estimation time interval is divided into successive overlapping time slots, referred to as the estimation windows. The ML estimation problem is solved at each estimation window to estimate the unknown parameters. The parameter estimates are then transferred to the next estimation window and used as initial estimates. The estimated FE model parameters and simulation error variances are directly transferred from one estimation window to the next and used as initial estimates. The estimated base acceleration time history over the estimation window is subdivided in two parts; the first part, which does not overlap with the next estimation window, is taken as final estimate. The second part, however, is transferred to the next estimation window and used as initial estimate for the next estimation sequence. Fig. 1 illustrates schematically the proposed sequential estimation approach. The estimation windows have a constant length (t_l = window length in number of time steps) and constant length overlap with the next window (t_o = length of overlap between two consecutive windows in number of time steps). The sliding (or moving) rate is defined as the difference (in number of time steps) between the starting point of two consecutive windows, i.e., $t_s = t_l - t_o$. The first part of the estimated base acceleration time history at the m^{th} estimation window, which does not overlap with the next estimation window, is denoted by $\hat{\mathbf{u}}^{g,m}_{t_1^m:t_1^m+t_s-1}$. Likewise, $\hat{\mathbf{u}}^{g,m}_{t_1^m+t_s:t_2^m}$ denotes the second part of the estimated base acceleration time history that is transferred to $(m+1)^{\text{th}}$ estimation window as initial estimate.

3. Numerical Validation Study

Numerically simulated structural response data obtained from a three-dimensional (3D) 5-story 2-by-1 bay reinforced concrete (RC) building frame structure subjected to bidirectional horizontal seismic excitation are used to verify the performance of the proposed parameter and input estimation framework (Fig. 1). A mechanics-based nonlinear FE model of the building frame, developed in OpenSees [9] is used to simulate the seismic response of the structure. In the simulation phase, the FE model of the structure and the bidirectional seismic input excitation are assumed to be known. The two horizontal ground acceleration records from the 2004 Parkfield earthquake (Cholame 2 west station) are selected as the seismic input excitation [10]. Simulated floor absolute acceleration responses at the first, fourth, and fifth (or roof) levels and the relative (to base) roof displacement response are contaminated with artificial measurement noise to represent the measured structural response quantities. In the estimation phase, the North-South (NS) component of the base excitation is assumed to be measured and known, while the East-West (EW) component is assumed to be unknown (i.e., unmeasured due to, for example, sensor malfunctioning). Moreover, a set of five FE model parameters characterizing the nonlinear material behavior of the concrete and reinforcing steel are treated as unknown parameters to be estimated. These parameters are: $\boldsymbol{E_c}$ = initial stiffness or Young's modulus of concrete, $\boldsymbol{f_c'}$ = concrete compressive strength, $\boldsymbol{E}$ = elastic (or Young's) modulus of reinforcing steel, σ_y = initial yield strength of reinforcing steel, and $\boldsymbol{b}$ = strain-hardening ratio of reinforcing steel. the true (exact) values of the FE model parameters are taken as $\boldsymbol{E}^{true} = 200\text{GPa}$, $\sigma_y^{true} = 400\text{ MPa}$, and $\boldsymbol{b}^{true} = 0.05$, $\boldsymbol{E}_c^{true} = 30\text{ GPa}$, and $\boldsymbol{f}_c'^{\,true} = 40\text{ MPa}$. The measured structural dynamic responses are utilized to estimate jointly the five FE model parameters and the full discrete time history of the base acceleration in the EW direction. The output-only nonlinear FE model updating approaches presented above is implemented in MATLAB [11] and interfaced with OpenSees for FE response and response sensitivity computations. More details can be found in [8].

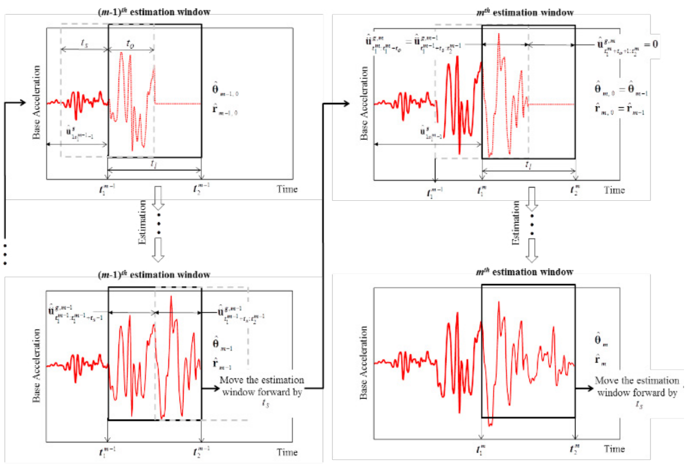

Fig. 1. Schematic representation of the proposed sequential estimation approach.

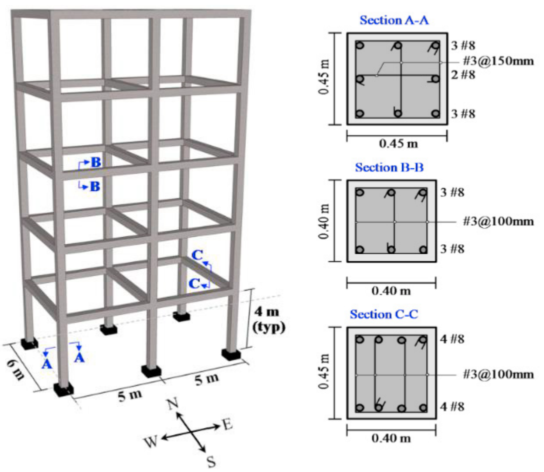

Fig. 2. RC building frame: isometric view and beam and column cross-sections.

The estimation window considered in this study has a length of 100 time steps (i.e., $\boldsymbol{t_l} = 100 \times 0.025 = 2.5$ sec) and a sliding rate of 50 time steps ($\boldsymbol{t_s} = 50 \times 0.025 = 1.25$ sec). By using this estimation window, the seismic input time history to be estimated, which is the first five seconds of the ground motion, is divided into three estimation windows.

The initial estimates of the FE model parameters ($\hat{\boldsymbol{\theta}}_0$) are arbitrarily selected as $\hat{\theta}_{0,1} = 0.80\boldsymbol{E}^{true}$, $\hat{\theta}_{0,2} = 1.40\sigma_y^{true}$, $\hat{\theta}_{0,3} = 1.20\boldsymbol{b}^{true}$, $\hat{\theta}_{0,4} = 0.80\boldsymbol{E}_c^{true}$, and $\hat{\theta}_{0,5} = 0.70\boldsymbol{f}_c'^{\,true}$. The initial estimates of the base accelerations are selected as zero. The feasible search domains for the FE model parameters and the base acceleration at each time step are selected as $0.5\boldsymbol{\theta}_0 \le \boldsymbol{\theta} \le 2.0\boldsymbol{\theta}_0$ and $-1.0\boldsymbol{g} \le \ddot{\boldsymbol{u}}^g \le 1.0\boldsymbol{g}$, respectively. The true value of the measurement noise variance for the absolute acceleration and relative displacement time histories, which are respectively polluted with 1% g and 5 mm RMS Gaussian white noises, are $\boldsymbol{r}_{Acc}^{true} = 9.6\times10^{-3}\,\mathrm{m}^2/\mathrm{s}^4$ and $\boldsymbol{r}_{Disp}^{true} = 2.5\times10^{-5}\,\mathrm{m}^2$. The initial estimate of the simulation error variance is selected as $\boldsymbol{r}_{Acc,0} = 2.2\times10^{-2}\,\mathrm{m}^2/\mathrm{s}^4$ for the acceleration response data and $\boldsymbol{r}_{Disp,0} = 9\times10^{-6}\,\mathrm{m}^2$ for the displacement response data, which corresponds to 1.5% g RMS and 3 mm RMS measurement noise, respectively. The feasible search domain for the simulation error variance vector is set as $0.1\hat{\mathbf{r}}_0 \le \mathbf{r} \le 10\,\hat{\mathbf{r}}_0$. The ML estimation problem, which is a nonlinear optimization, is solved using an interior-point method [12] in this study, which is available as a part of the MATLAB optimization toolbox. The convergence criterion for the optimization algorithm is defined by two conditions. The optimization process is considered converged if any of the following two conditions are satisfied.

$$\text{Condition 1:}\ \left\| \begin{bmatrix} \hat{\boldsymbol{\psi}}_m \\ \hat{\mathbf{r}}_m \end{bmatrix} - \begin{bmatrix} \hat{\boldsymbol{\psi}}_{m-1} \\ \hat{\mathbf{r}}_{m-1} \end{bmatrix} \right\|_2 \le 10^{-4},\ \text{Condition 2:}\ \left\| \nabla \boldsymbol{J}(\boldsymbol{\psi},\mathbf{r}) \right\|_\infty \le 10^{-4} \tag{7}$$

where $\hat{\boldsymbol{\psi}}_m$ is the normalized estimated parameter vector at the m^{th} optimization iteration, in which each component of the estimated parameter vector is normalized by the corresponding initial estimate.

Fig. 3 compares the true and estimated EW component of the base acceleration time history. The estimation error time history, which is defined as the difference between the true and estimated base acceleration time histories, is also shown in this figure. As can be seen, the EW component of the base acceleration time history is very well estimated. The base acceleration at the last few time steps is not estimated correctly due to insufficient pertinent information in the measured response of the structure.

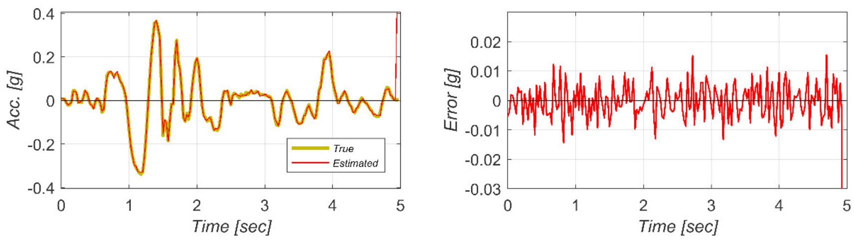

Fig. 3. Left: Comparison of the true and estimated base acceleration time history in the EW direction; Right: estimation error time history

Table 1 compares the final estimates of the five unknown FE model parameters normalized by their corresponding true values. As can be observed, the model parameters are estimated with good accuracy. This table also reports the relative root mean square error (RRMSE) of the estimated base acceleration time history in the EW direction. The RRMSE is defined in the following equation. The small RRMSE validates the correct performance of the presented estimation framework.

$$\text{RRMSE}\left(\hat{\ddot{\mathbf{u}}}_{1:k}^{g}\right)(\%) = \sqrt{\frac{\sum_{i=1}^{k}\left(\hat{\ddot{u}}_i^{g} - \ddot{u}_i^{g,true}\right)^2}{\sum_{i=1}^{k}\left(\ddot{u}_i^{g,true}\right)^2}} \times 100 \tag{8}$$

Table 1 also reports the estimated simulation error variances for the eight measurement channels, normalized by the corresponding true measurement noise variance. As can be observed the estimated simulation error variances are not as accurate as the model parameter estimates. As shown in [8], the estimation accuracy of the simulation error variance depends on the number of time samples of the measured data used in the estimation. Therefore, by increasing the length of the estimation window, the estimation accuracy of the simulation error variances would be increased.

However, increasing the length of the estimation window would increase the number of iterations and, therefore, the computational cost of the estimation algorithm.

Table 1. Comparison of the estimated FE model parameter and the simulation error variances.

Final estimates of material parameters					Final estimates of simulation error variance, r / r^{true}								RRMSE ($\hat{\mathbf{u}}^g$) (%)
$\hat{E}/E^{true}$	$\hat{\sigma}_y/\sigma_y^{true}$	$\hat{b}/b^{true}$	$\hat{E}_c/E_c^{true}$	$\hat{f}'_c/f'^{true}_c$	Acc1 EW	Acc1 NS	Acc4 EW	Acc4 NS	Acc5 EW	Acc5 NS	Disp5 EW	Disp5 NS	
1.00	0.99	1.02	1.00	1.00	2.08	1.17	0.63	0.90	0.63	0.96	0.80	0.79	5.24

4. Conclusions

This paper presented a framework for output-only nonlinear system and damage identification of civil structures. This framework was based on nonlinear finite element (FE) model updating using only the measured structural response to earthquake excitation. A sequential maximum likelihood (ML) estimation approach was proposed to jointly estimate the unknown FE model parameters, the input ground acceleration time histories, and the simulation error variances. In this approach, the estimation time interval was sub-divided into successive overlapping estimation windows. The estimation problem was solved sequentially over all the estimation windows and at each estimation window, it was solved iteratively to produce a set of estimation results that were transferred to the next estimation window. The sequential maximum ML estimation method reduced to a sequential constrained nonlinear optimization approach, which required the computation of FE response sensitivities with respect to the unknown FE model parameters and the base acceleration time history. The FE response sensitivities were computed accurately and efficiently using the direct differentiation method (DDM). A numerical study validated the performance of the proposed framework in estimating accurately the unknown FE model parameters and the input ground acceleration time history.

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